ABSTRACT.

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Thesis title: Analysis of a suspension-damping model.

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Abstract: Classically, the motion of a system consisting of a spring with a fixed end and attached to a rigid moving mass at the other, has been modelled by the classical second order ODE $mu''(t) + du'(t) + ku(t) = 0$. But phenomenons such as internal deformation differences or internal viscous damping are not taken into account by this model. That is why partial differential equations models arise. In this thesis, we propose and justify a model for those viscoelastic systems, which turns to be a *wave equation with strong damping* (or Kelvin-Voigt damping) and *dynamical boundary conditions*. We analyze this model in terms of two parameters: $\alpha \geq 0$, the spring internal viscosity, and $\varepsilon \geq 0$, which essentially is the inverse of the moving mass at the end. The main purpose will be to compare this continuous approach with the classical model and to see in which case the PDE admits an ODE as limit, in an appropriate sense. The tool used to prove this are the *dominant eigenvalues*, so that a detailed analysis of the spectrum (including eigenvalues and essential spectrum) allows us to show the nonexistence of a limit ODE for a purely elastic spring ($\alpha = 0$), the existence of a nonuniform limit ODE when the internal viscosity is small ($\alpha \sim 0$) and the existence of a limit second order ODE, which is given explicitly, when the mass at the end is taken sufficiently large ($\varepsilon \sim 0$).

Another problem of interest is obtained by imposing an acceleration in the previous fixed end. This point of view, which can also be thought as an external control, gives rise to the previous model but with a nonlocal nonlinearity in the equation and in the boundary conditions. With the purpose of showing the existence of a limit ODE for this nonlinear problem, we prove the existence of an *exponentially attracting invariant manifold* for ε sufficiently small, which converges to 0 in the $\mathcal{C}^1$ topology when $\varepsilon \to 0$. This is used to find explicitly a limit second order nonlinear ODE. In this part, the use of perturbation theory tools such as the convergence of operators in a generalized sense or a uniform bound for families of semigroups are essential.