Mathematical model of rowing in catalan *llagut*

M. Aguareles, X. Mora, M. Noguera, M. Pellicer.

1 Introduction

The catalan *llagut* is a traditional boat that nowadays is used in Catalonia in local rowing competition. It is similar to the boat used in classical Olympic rowing, but with a main and important characteristic that makes both ways of rowing substantially different: the seats are fixed and cannot move, while in an olympic rowing boat the seats move. This is why the movement of the boat is achieved by bending the torso rather than bending the legs as it is the case in Olympic rowing.

The rowing in catalan *llagut* is not a very well understood mechanism. Several different problems were proposed in the GEMT, but we concentrated on two main aspects (developed in the sections below): the stroke efficiency and the effect of the rower movement on the boat in terms of inertia. As there are no models of the catalan *llagut* rowing, our approach is based on the work of [1], where the modelling of a sliding seat boat is developed. Despite of the differences between the two types of rowing that we have pointed out before, some parts of the work of [1] can be adapted and used in our case. In some other works, like for instance [2] or [3], the very important issue of the design and performance of the oar's blade is considered. We will not focus on this issue, on the contrary, in this work we are concerned with a more general approach to give some insight into the rower's performance in fix-seat rowing competition.

The report is structured as follows. In section 2 we analyze an aspect of the stroke efficiency which is the angle of catch in which the oar is inserted into the water. That is, without taking into account limitations coming from the rower, we wonder which is the best angle to insert/take out the oar into/from the water in order to miss as little energy as possible. In section 3 we face a second aspect on the efficiency of the stroke that is the relation between the force made onto the oar and the resulting velocity of the boat. Again, we do not take into account restrictions due to the rower. In section 4 we change our point of view considering only the rower movement and not the oar, and we study its effect onto the velocity of the *llagut* by analyzing the forces that are created. This is important in order to understand the recovery part of the rowing (the part in which the oar is taken out from the water and placed back in its catch position). We try to verify that a smoother recovery is better than a quicker but sudden one.

This report has been written based on the contributions from the discussions at the Grups d'Estudi, that included the following members: Maria Aguareles, Xavier Alameda, Enric Fossas, Antoni Guillamon, Albert Mas, Xavier Mora, Miquel Noguera, Marta Pellicer, Peregrina Quintela and Noemí Ruiz.

2 Aquatic phase: angles of catch and finish

It is important for the rower to insert the oar at the right time so that no missed or splashed water occurs. As it is pointed out in [1], the way to achieve such an optimal stroke is by inserting or extracting the blade of the oar into or from the water in the precise moment in which the blade's absolute velocity has the same direction as the oar (see figure 1). This way the insertion and the extraction of the blade into and from the water, which is a movement in the vertical plane perpendicular to the water, is done so that the relative velocity of the blade with respect to the water is purely in this direction. This means that the insertion and extraction becomes as smooth as possible and as a consequence the amount of energy that is lost in this process is as small as possible. Of course this condition is independent of the way the rowing is done, that is to say, it does not depend on whether the seat moves or not. What will make the difference is the way the rower moves in order to achieve the corresponding angles of catch and finish, which may induce a different inertia moment on the boat and might therefore affect the resulting boat velocity. The effect of the inertia of the rower into the boat is discussed in section 4.

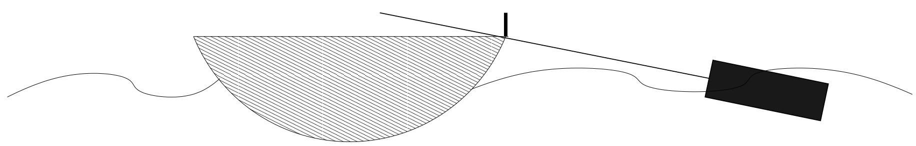

Figure 1: Cross-section of the *llagut* with one oar.

The absolute velocity of the oar, $\mathbf{v_o}$ depends on the velocity of the boat, $\mathbf{v_b}$ and also on the relative velocity of the oar with respect to the boat $\mathbf{v_{o/b}}$ through the relation

$$\mathbf{v_o} = \mathbf{v_b} + \mathbf{v_{o/b}}. \tag{1}$$

In a reference system fixed on the boat, the movement of the blade is purely radial, since its length is fixed and we assume, as a first approximation, that the oar is rigid and cannot bend. Therefore, the velocity of the oar (or the blade) with respect to the boat, $\mathbf{v_{o/b}}$ has the direction perpendicular to the oar itself. Hence, the non-splashing condition does actually say that the absolute velocity of the oar $\mathbf{v_o}$ must be perpendicular to the relative velocity of the oar with respect to the boat $\mathbf{v_{o/b}}$. Then, writing all the velocities in terms of the angle of the oar with respect to the boat (see figure 2), one finds that

$$\mathbf{v}_b = \pm v_b \cos\phi \, \mathbf{e}_\phi + v_b \sin\phi \, \mathbf{e}_r \tag{2}$$

$$\mathbf{v}_{o/b} = v_{o/b} \, \mathbf{e}_\phi \tag{3}$$

where $\mathbf{e}_r$ and $\mathbf{e}_\phi$ stand for the unitary vectors in the direction of the oar and in the direction perpendicular to the oar respectively. Upon using (1) the absolute velocity of the oar reads

$$\mathbf{v}_o = (\pm v_b \cos\phi + v_{o/b}) \, \mathbf{e}_\phi + v_b \sin\phi \, \mathbf{e}_r,$$

so the non-splashing/non-missing condition becomes

$$\pm v_b \cos\phi + v_{o/b} = 0, \tag{4}$$

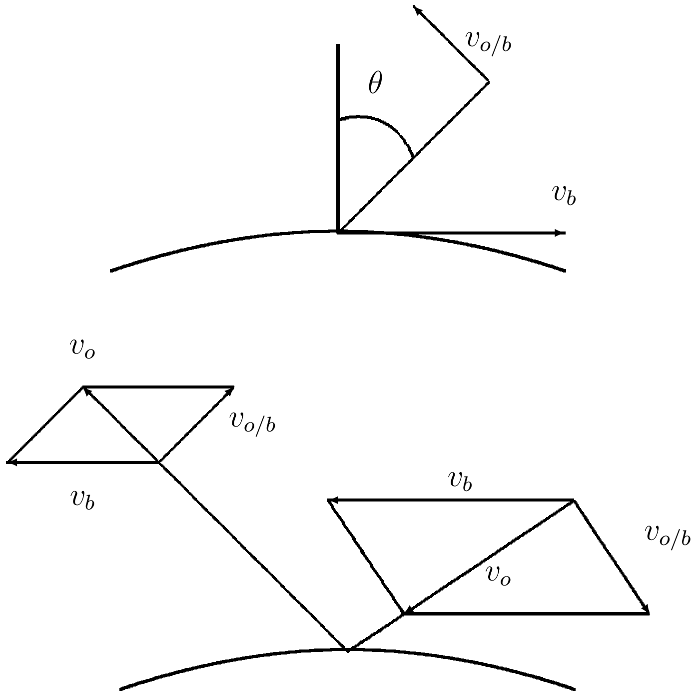

Figure 2: Velocities of the oar, the boat and the oar with respect to the boat by means of the angle of the oar with respect to the boat.

and thus, while the blade is inside the water $\mathbf{v}_o \cdot \mathbf{e}_\phi > 0$. Hence the optimal angles of the oar with respect to the boat to use to put it into the water and to take the blade out are

$$\cos\phi_{max/min} = \pm\frac{v_{o/b}}{v_b}. \tag{5}$$

From the experience of the rowers it is commonly accepted that the best way to row is by inserting the blade at an angle that is smaller than the angle of extraction. One of the questions that was asked to the GEMT was if this way of rowing had actually any theoretical base that supported it. The analysis of the angles shows that this way of rowing is actually consistent with condition (5) due to the fact that when the blade is about to be taken out, the rower is in such a position that he or she is able to perform the maximum strength and therefore the relative velocity of the oar with respect to the boat is also greater, which produces ϕ_{max} to be greater than ϕ_{min}. As the boat accelerates and v_b becomes larger, the angle of catch can be smaller, while the finish angle at which the blade is taken out can be kept large due to the fact that the rower can also move the oar faster (the opposition of the water is weaker).

3 The relation between oar force and velocity

In this section we discuss the way the force that the oar applies into the water changes the boat's velocity. We again focus on the results in [1] and adapt conveniently the expressions to the fixed-seat rowing modality.

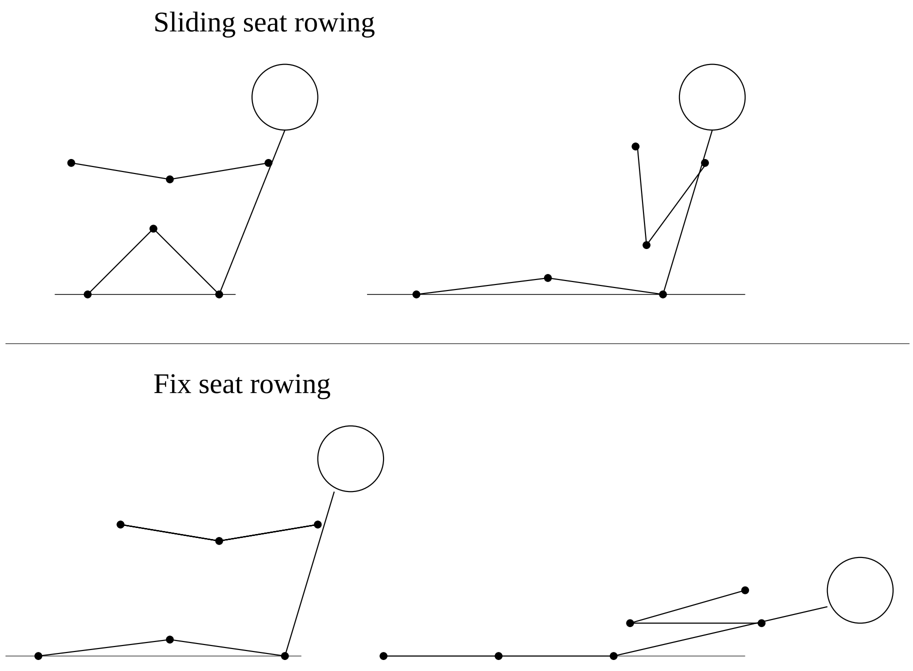

Figure 3: Fixed seat and sliding seat rowing modality, sketch of the rower's movement.

We now write a one-dimensional model for the movement of the boat where we do not consider effects like the pitch of the boat as it moves, which in a more refined model should definitely be taken into account. This pitch effect is related to the analysis of the inertia driven by the rowers as they bend their torso up and down, which is analyzed in section 4, but it is also strongly dependent on of the sea waves. It is important to remark that this pitch effect, that in an sliding seat competition is not important, may become crucial when considering fixed seat rowing in the sea.

We use the same notation as in [1]: F_{hand} stands for the force that the rower applies to the oar, $-F_{foot}$ is the propulsion force that the rower does with his or her feet and abdominal muscles in order to move his or her torso backwards (see figure 3 for a sketch of the rower's movement), F_{lock} is the force that the oar applies on the boat through the oarlock, F_{boat} is the drag force of the boat in the water, $-F_{oar}$ is the force that the oar applies onto the water (and θ is the oar angle in a plane parallel to the water), which has the direction perpendicular to the blade. We also denote by x_R and m_R the position of the centre of gravity of the rower and his or her mass, x_b and m_b are the position of the centre of gravity of the boat and its mass, and x_o and m_o are the position of the centre of gravity of the oar and its mass (see figure 4). Having all these magnitudes defined, we follow [1] to write the force balance equations for the rower, the oar and the boat, that we write here for a complete understanding of the text:

$$-F_{hand} + F_{foot} = m_R \ddot{x}_R, \tag{6}$$

$$F_{hand} - F_{lock} + F_{oar} \cos\theta = m_o \ddot{x}_o, \tag{7}$$

$$-F_{boat} + F_{lock} - F_{foot} = m_b \ddot{x}_b. \tag{8}$$

The positions of the oar and the rower can be expressed in terms of x_b, the position of

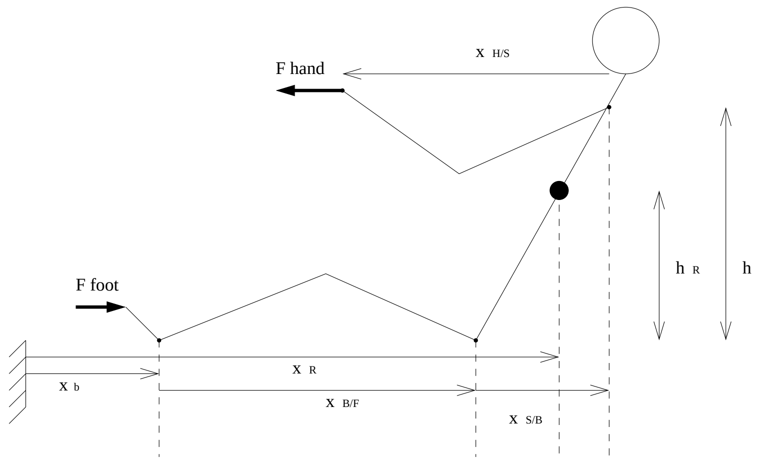

Figure 4: Representation of some magnitudes involved in the modelisation.

the boat centre of mass:

$$x_o = x_b + d_{L/F} + d\sin\theta,$$
$$x_R = x_b + x_{B/F} + r x_{S/B}$$

where $d_{L/F}$ is the horizontal distance between the oarlock and the rower's feet, d the distance between the oarlock to the oar's centre of mass. As for $x_{B/F}$ and $x_{S/B}$, they represent the distance between the seat and the feet (which is fixed in this case) and the distance between the seat and the shoulders (see figure 4). Finally, r stands for the ratio of the rower's center of mass height (h_R) to the shoulder's height (h_s), both measured with respect to the seat. These relations can be differentiated twice to find the corresponding relation between the accelerations in terms of $\ddot{x}_b$, obtaining

$$\ddot{x}_o = \ddot{x}_b + d\ddot{\theta}\cos\theta - d\dot{\theta}^2\sin\theta, \tag{9}$$
$$\ddot{x}_R = \ddot{x}_b + r\ddot{x}_{S/B}. \tag{10}$$

The drag force due to the water in this type of problems is usually considered as

$$F_{boat} = C_1\dot{x}_b^2, \tag{11}$$

where C_1 is the boat drag coefficient which must be determined from drag tests. Hence, combining equations from (6) to (11) the following second-order differential equation is obtained

$$(m_R + m_b + m_o)\ddot{x}_b = -C_1\dot{x}_b^2 + F_{oar}\cos\theta - m_R r\ddot{x}_{S/B} - m_o d(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta). \tag{12}$$

We are going to simulate the rower's performance in one stroke using this equation, considering x_b unknown. However, in order to do this in a more realistic way, we must approach some other terms in the equation conveniently.

As for F_{oar}, we use as a very first approach the one given in [1]

$$F_{oar} = C_2\left(l\dot{\theta} + \dot{x}_b\cos\theta\right)^2. \tag{13}$$

Now, C_2 is the coefficient for the blade force and l stands for the length of the oar that is outside the boat. It has to be said that a better modelling of F_{oar} in the case of the rowing in the sea and with a *llagut* might be considered in a future work.

On the other hand, the angle that the oar forms with respect to the boat is related to the rower's coordination by

$$d_{L/F} - s\sin\theta = x_{B/F} + x_{S/B} - x_{H/S}$$

where s stands for the length of the oar that is inside the boat, that is to say, the distance between the oarlock and the rower's hands (see [1] for details). As for $x_{H/S}$, it represents the distance between the hands and the shoulders (see figure 4). Hence, differentiating the above expression twice with respect to time we obtain

$$s\ddot{\theta}\cos\theta = \ddot{x}_{S/B} - \ddot{x}_{H/S} + s\dot{\theta}^2\sin\theta \tag{14}$$

since, in our case, $x_{B/F}$ is fixed. Using equations (12), (13) and (14) we now simulate the rower's performance in one stroke, meaning that we focus on the evolution of several magnitudes from the catch time until the rower finishes the stroke.

The rower's movement under consideration and the hypothesis that have been used for the simulations are as follows:

1. The distance between the feet and the seat is constant, $x_{B/F} = 1m$ (the seat does not slide).

2. The rower is initially bent forward with his or her back at an angle of 2/3 radians and his or her arms are completely stretched. Then the rower moves backwards until he or she lies at an angle also of 2/3 radians, still keeping the arms stretched. The angle of the body is taken as a linear function of time. This movement takes one second.

3. After this second, keeping the backs inclination, the rower tucks his or her arms. This is modelled by using a linear function in time for $x_{H/S}$ that changes in 0.5 seconds from 0.75 metres to zero.

4. The length of the back is assumed to be $h_S = 0.6m$.

As initial condition we assume that the boat is already moving at a velocity of 3 m/s, so we are actually simulating the effect of one stroke when the boat has been already accelerated, that is to say, after the competition has started.

With these hypotheses, we obtain an approximated F_{oar} from equation (13), and an approximated θ from equation (14). Then, we use equation (12) with these approximations and obtain the corresponding rower's performance.

The results are shown in figure 5. The upper pictures show that the velocity of the boat increases when the rower tucks his or her arms, which is due to the fact that the oar's force onto the water does also increase very much in this period, as can be seen in the bottom left picture. As for the evolution of the oar's angle (bottom right picture), we find that when the rower performs this movement the oar is moved slower the first second, but then it is moved faster before the stroke finishes. The strokes are very often performed this way, since this last pulling of the oar helps the rower to achieve the inertia to then stand up and start the next stroke. But the picture for the velocity (upper right) also shows that by doing this non-smooth movement the velocity at some point decreases, although at the end of the stroke a higher velocity is achieved. Therefore it would be interesting to model some other rower's performance that was smoother to see whether the boat achieved a higher velocity at the end of the stroke. In section 4 we will see how this non-smooth movement affects the velocity of the boat.

This analysis does only consider the phase of the period where the oar is inside the water. A complete analysis should couple equation (12) with a model of the forces and velocities when the oar is outside of the water. This second analysis should take into account the rower's change in his or her body's acceleration.

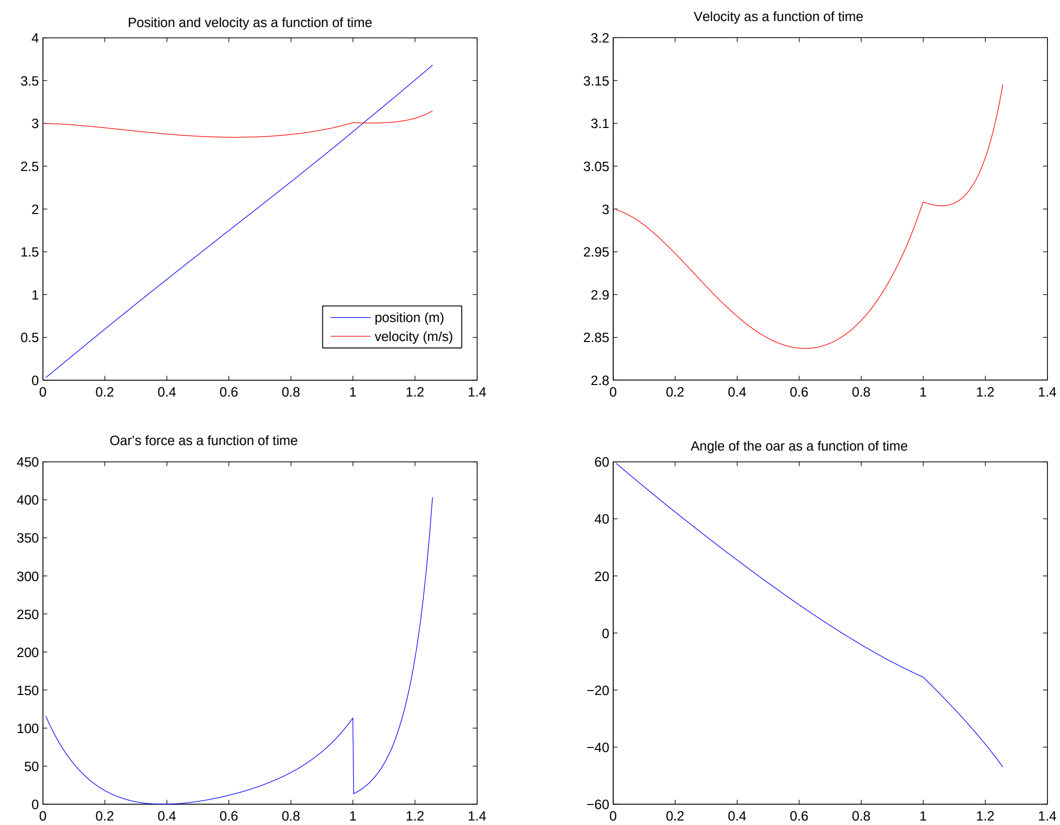

Figure 5: Upper pictures: (left) evolution of the velocity and position of the boat, (right) velocity of the boat. Bottom pictures: (left) oar force, (right) oar angle.

4 The effect of the rower movement onto the *llagut* velocity

In this section we are concerned with the influence of the rower movement on the boat displacement. Thus, from now on we will imagine a *non-rowing rower*, that is, a rower without oars but only moving himself or herself in a certain given way. Moreover, we will think of the rower as a unique point with its centre of gravity moving in a line (forward and backwards).

Although it may seem strange, this simplification makes sense since we are concerned with a better understanding of how the recovery movement has to be done, which one of the questions we were asked for. The recovery is the aerial phase of the stroke, when the oars are taken out from the water and the rower bends *forward* (opposite to the advance of the boat) in order to be ready for the next stroke. Since during this process the oars are not making any force to progress, the rower has to do this movement as quick as possible. But doing it too quickly introduces an acceleration against the inertia of the boat. Consequently, we have to achieve a certain balance between both factors.

The relation between the rower movement and the boat displacement is given by equation (17). We are going to explain this model, which also follows from the one given in [1] when the present assumptions are used.

If we denote by x_R the absolute position of the rower and by m_R his or her mass, the linear momentum balance for the rower gives:

$$m_R \ddot{x}_R = -F \tag{15}$$

where F stands for the force that the rower is doing against the boat. Denoting by x_b the absolute position of the boat and by m_b its mass, the linear momentum balance for the boat gives:

$$m_b \ddot{x}_b = F - C_1 \dot{x}_b |\dot{x}_b| \tag{16}$$

where C_1 is the boat drag coefficient. Observe that, since in this part we may consider forward and backward movement of the boat, and that since the drag force always acts against this, we have to consider $\dot{x}_b|\dot{x}_b|$ instead of $\dot{x}_b^2$ as in the previous section. Here we assume that the only forces acting on the boat are the reaction force from(15) and the drag force, which is typically taken in the form given in equation (16). Note that in both equations the oars do not have any role.

We now consider $x_{R/b}$ as the position of the rower relative to the boat, that is:

$$x_{R/b} = x_R - x_b.$$

This new variable allows to combine equations (15) and (16) into the following one, that determines the motions of the boat when the motion of the rower is given:

$$m_b \ddot{x}_b = -C_1 \dot{x}_b |\dot{x}_b| - m_R \ddot{x}_{R/b}. \tag{17}$$

Our aim is to check with some examples whether a too quick recovery movement has a negative effect, as it seems to be commonly accepted. Using equation (17), we now look at the distance covered by the boat for different kinds of rower actions. More specifically, we have computed this distance during a fixed interval of time for four kinds of rower actions.

(a) **Symmetrical catch and finish.** We consider a rower doing the recovery movement at the same velocity as the catch one, that is, moving forward and backwards at the same velocity. More particularly, we have considered:

$$x_{R/b}(t) = \sin\left(\frac{2\pi t}{T}\right)$$

where T is given.

(b) **Slow catch, fast recovery.** This has been modelled by the function:

$$x_{R/b}(t) = \sin\left(\frac{2\pi t}{T}\right) - 0.25\sin\left(\frac{4\pi t}{T}\right).$$

(c) **Fast catch, slow recovery.** This has been modelled by the function:

$$x_{R/b}(t) = \sin\left(\frac{2\pi t}{T}\right) + 0.25\sin\left(\frac{4\pi t}{T}\right).$$

(d) **Still rower.** We are interested in comparing the displacement obtained with the fast/slow recovery movements with the one obtained when the rower makes no action, that is:

$$x_{R/b}(t) = 0.$$

We wonder if it could be even better (in terms of achieving a larger displacement of the boat) doing nothing rather than doing a bad movement.

Also, we have tried these four kinds of $x_{R/b}$ in two cases: a boat moving with a certain initial velocity and a boat initially at rest. The distances covered by the boat in each case can be seen in figure (6). As the rower is facing the back part of the boat, the catch movement takes place when $\dot{x}_{R/b} > 0$, while the recovery part is when $\dot{x}_{R/b} < 0$. The slopes of $x_{R/b}$ in each part give the velocity at which each movement is done.

As we can observe in the graphics, there is no difference (qualitatively speaking) between the case of a boat with initial velocity and a boat initially at rest (apart from the last one, as it is obvious). The only difference is the total covered distance, as it should be expected.

However, the kind of rower action has a significative effect. We can see that in case (c) the boat goes forward all the time. However, in case (b) eventually moves backwards!

Remark 4.1. *The amazing fact that (c) eventually moves backwards is a consequence of the quadratic character of the friction term. If we assumed a linear damping term (that is, $m_b\ddot{x}_b = -C_1\dot{x}_b - m_R\ddot{x}_{R/b}$) we would observe that there would not be any difference between the solutions, qualitatively speaking. In this case the difference would only be the total covered distance (see figure (7).*

After these examples, the nowadays accepted opinion among some fix-rowing coatchs that a quick recovery reduces the efficiency of the stroke seems to be correct. It even seems that a rowing technique following case (d) would be better than the one following (b). But in the real problem this recovery phase is combined with the catch one (which includes the oar forces and so the equation that models the complete rowing is not (17). Moreover, in a more realistic approach we should take into account that the recovery has to be done with a certain velocity in order to start the next stroke with strength enough. This could be the subject of a future work. Anyway the results of this section are a strong evidence in favour of the fast catch-slow recovery strategy.

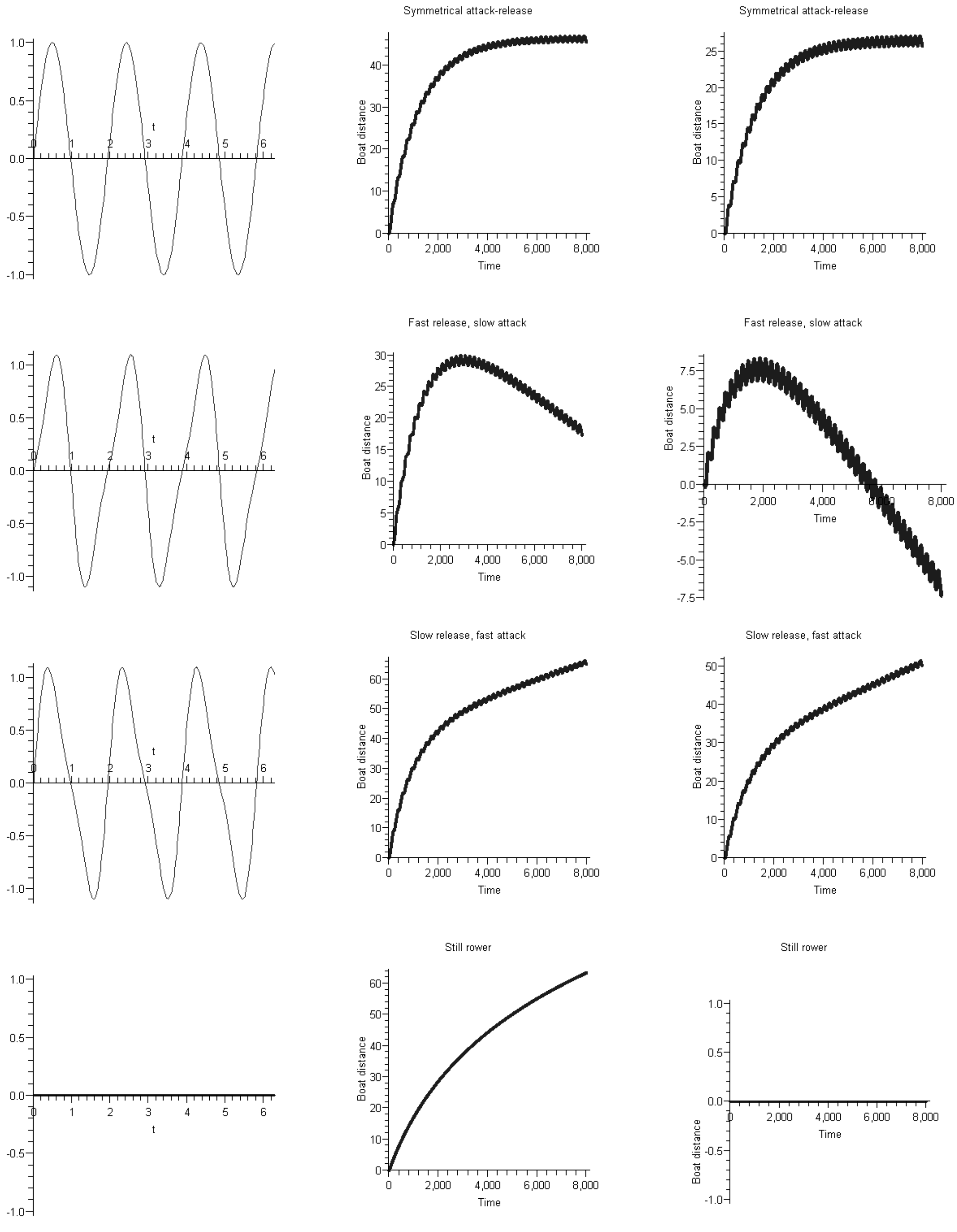

Figure 6: (Left) position of the rower's centre of gravity, (centre) displacement of the boat with initial velocity, (right) displacement of the boat initially in rest for the four types of rower movement.

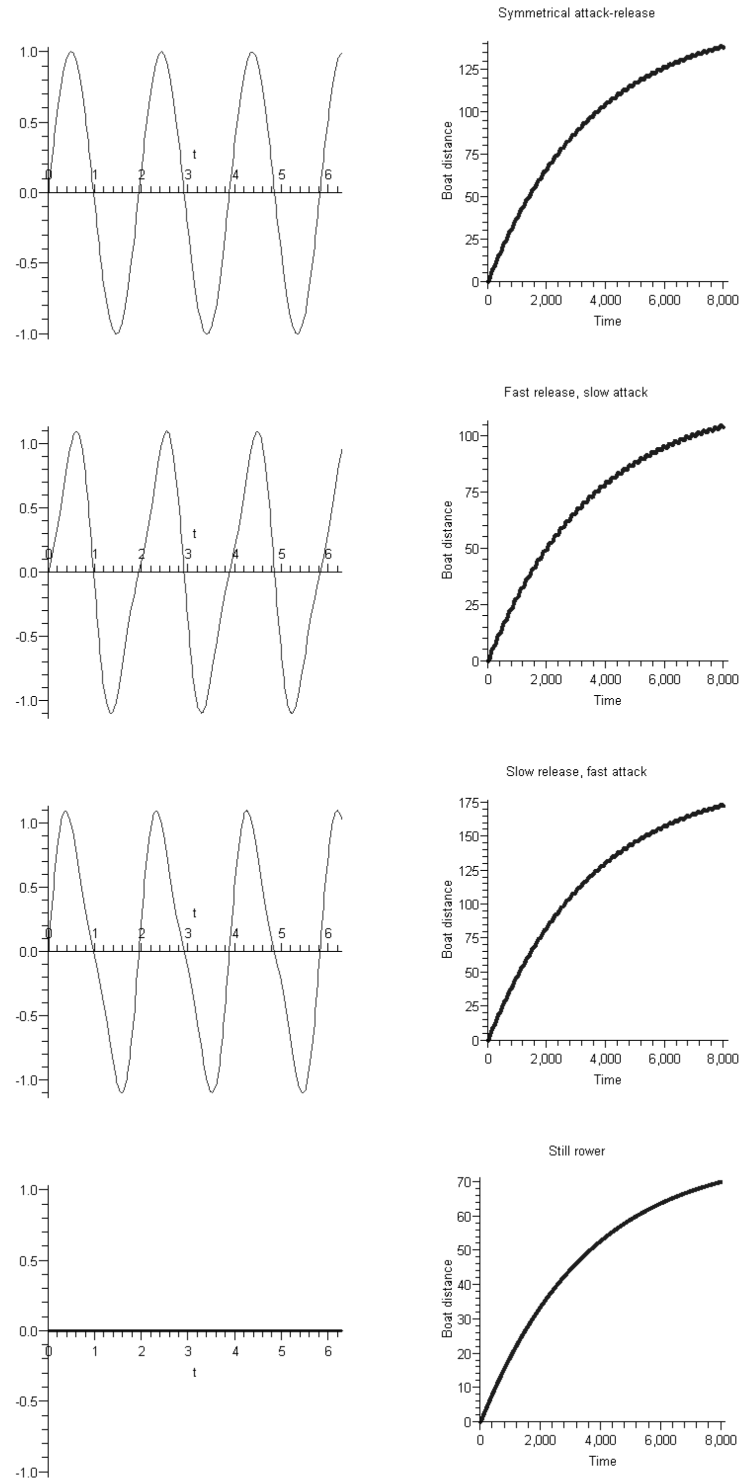

Figure 7: (Left) position of the rower's centre of gravity, (right) displacement of the boat with initial velocity for the four kinds of rower movement in the case of a linear damping.

References

[1] D. Cabrera, A. Ruina and V. Klevshnev, *A simple 1+ dimensional model of rowing mimics observed forces and motions.* Human Movement Sciences. **25** (2006), 192-220.

[2] N. Caplan, and T. Gardner, *A fluid dynamic investigation of the big blade and macon oar blade designs in rowing propulsion.* J. Sports Sci. (2007 Apr) **25 (6)**, 643-50.

[3] N. Caplan, and T. Gardner, *A mathematical model of the oar blade - water interaction in rowing* J. Sports Sci. (2007 Jul) **25 (9)**, 1025-34.

[4] D. L. Pope, *On the dynamics of men and boats and oars.* Mechanics and Sport. ASME, pp 113-130.