Comparing hemisphere sampling techniques for obscurance computation

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Abstract

Obscurance method is a fast and simple way to mimic diffuse illumination in a 3D virtual environment. This method requires to take samples over the hemisphere to compute the Monte Carlo integral of the obscurance function. The efficiency of the obscurance computation is thus related to the sampling technique used. We compare in this paper four different sampling techniques: uniform random, quasi-Monte Carlo with random offset, systematic and stratified sampling.

Keywords: Computer Graphics, Three-Dimensional Graphics and Realism, Ray-tracing, Radiosity, Sampling Techniques.

1. Obscurances

Obscurances [19, 6, 10, 11] take account of secondary diffuse illumination, being totally decoupled from direct illumination. Obscurance for point P is defined as:

$$W(P) = \frac{1}{\pi}\int_{\omega\in\Omega} \rho(d(P,\omega)) \cos\theta d\omega \qquad (1)$$

where

- $\rho(d(P,\omega))$: function with values between 0 and 1, and giving the magnitude of ambient light incoming from direction ω
- $d(P,\omega)$: distance from P to the first intersected point in direction ω
- θ: angle between direction ω and the normal at P

This integral is computed using Monte Carlo integration, by randomly sampling cosine weighted directions over the hemisphere, and averaging the obscurances function for the obtained samples.

A simplified version of the obscurances technique is used under the name of *ambient occlusion* in several commercial renderers, such as *Photorrealistic Renderman* by *Pixar* [9, 1].

2. Hemisphere sampling

To obtain a cosine weighted random direction over the hemisphere we sample twice a random variable uniformly distributed in the unit interval (i.e. calling the *drand()* function), obtaining ξ_1 and ξ_2. The direction (ϕ,θ) is then given by:

$$\begin{cases} \phi = 2\pi\xi_1 \\ \theta = \arcsin\sqrt{\xi_2} \end{cases} \qquad (2)$$

Instead of using a pseudo-random number generator to obtain ξ_1 and ξ_2, we can use deterministic or quasi-Monte Carlo sequences, like Halton one [13]. Quasi-Monte Carlo sequences distribute the samples more regularly over the domain. To avoid bias, the values in the sequence can be added to the same random offset (Cranley-Patterson rotation [8]).

2.1. Systematic Sampling

Systematic sampling [7, 4, 18] is a classical Monte Carlo technique that has been used for years in some fields, notably in Stereology [2, 3, 5]. In systematic sampling a uniform grid is translated by a random offset giving the sampling points to probe the target function. As systematic sampling is based on regular sampling, we obtain cheaper samples than those obtained with independent uniform random sampling. It can be proved that for certain kind of functions the variance in systematic sampling decreases faster than in pure Monte Carlo sampling [2]. The drawback is that systematic sampling produces systematic error when the domain is somehow regular.

To take n_1 x n_2 systematic samples on the hemisphere, we proceed as follows [5]. We consider first n_1 x n_2 partitions of the hemisphere, and take $t_1 = \frac{2\pi}{n_1}$ and $t_2 = \frac{1}{n_2}$ as the periods for longitude and latitude respectively. We sample twice a random variable uniformly distributed in the unit interval, obtaining ξ_1 and ξ_2.

The directions are then computed as follows:

$$\begin{cases} \text{for } i = 0 \ldots (n_1 - 1) \\ \qquad \phi_i = t_1(\xi_1 + i) \\ \text{for } j = 0 \ldots (n_2 - 1) \\ \qquad \theta_j = \arcsin \sqrt{t_2(\xi_2 + j)} \end{cases} \quad (3)$$

2.2. Stratified Sampling

Stratified sampling [17, 16] is based on this same idea of dividing the domain into subdomains of equal probability, but the samples are chosen in a different way. Instead of choosing a random offset, we get a random sample from each subdomain. For a certain kind of functions, the variance of stratified sampling with one sample per stratum also decreases faster than in pure Monte Carlo sampling [17]. To take n_1 x n_2 stratified samples on a hemisphere, we proceed as follows.

We sample n_1 x n_2 pairs of random variable uniformly distributed in the unit interval, obtaining pairs (ξ_i and ξ_j), $i = 1, \ldots, n_1$, $j = 1, \ldots, n_2$. The directions are then given by:

$$\begin{cases} \text{for } i = 0 \ldots (n_1 - 1) \\ \qquad \phi_i = t_1(\xi_i + i) \\ \text{for } j = 0 \ldots (n_2 - 1) \\ \qquad \theta_j = \arcsin \sqrt{t_2(\xi_j + j)} \end{cases} \quad (4)$$

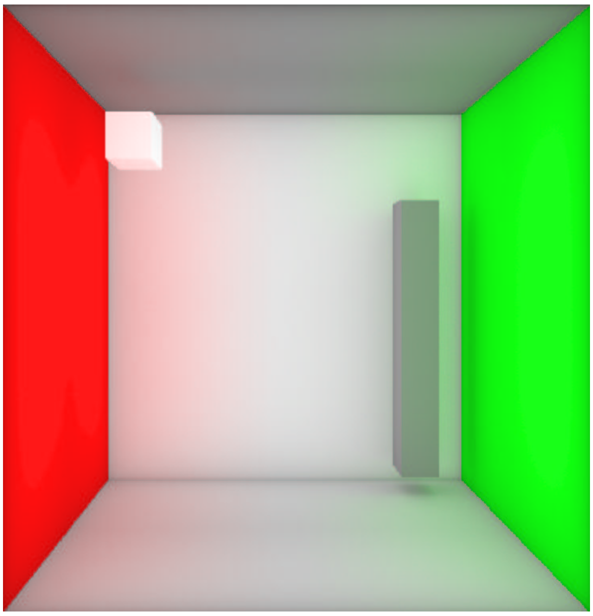
3. Results

We have used two scenes, kitchen and box (see fig. 1), and a series of five sample numbers, 2x2, ..., 6x6, to test uniform random, systematic, stratified and Halton sampling. This last one is done with random offset, as used in [12], because pure Halton sampling results in an unacceptable pattern. In fig. 2a and 2b we present charts of Mean Square Error (MSE) versus time, for the average of 10 executions. From these graphs we see that both systematic and stratified perform very similar and overcome uniform random. However, the systematic nature of the error in systematic sampling (see fig. 3) is an important drawback of this technique. We see also from fig. 2a & 2b that Halton sampling with random offset is the most efficient technique. See fig. 4 for a visual comparison of all four methods.

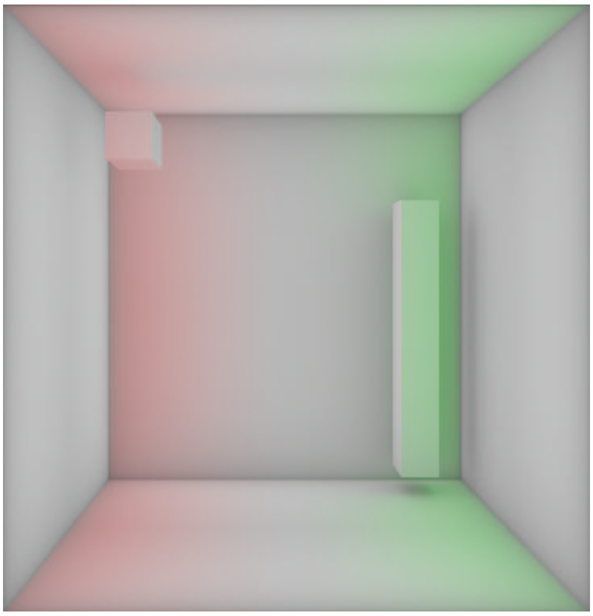
4. Conclusions and future work

We have compared in this paper four sampling methods to compute obscurances, uniform sampling, systematic, stratified and Halton with random offset. Both systematic and stratified have shown an improvement of 50% in efficiency with respect to uniform sampling, although systematic sampling presents a highly irregular distribution of the error. Halton sampling with random offset has resulted in still a higher improvement.

As the error in computing the obscurances is unevenly distributed (see fig. 5), we plan in the future to use an adaptive sampling strategy, this is, using more samples where they are more needed, according to some oracle function (variance [14], f-divergences [15]). In this strategy, batches of samples are progressively cast and the homogeneity of the obtained obscurances is examined. An heuristic is then devised to continue or stop sampling.



(a.i) Indirect illumination of Box scene.

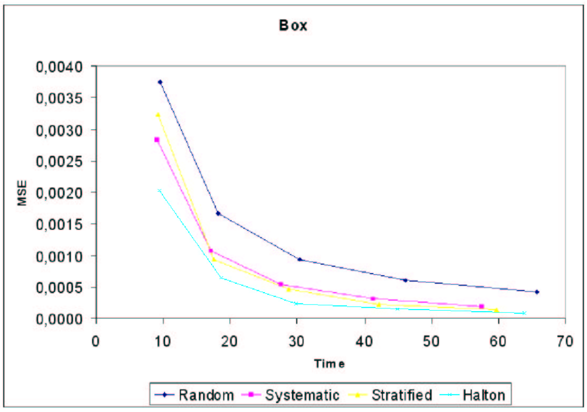


(a.ii) Obscurances of Box scene.

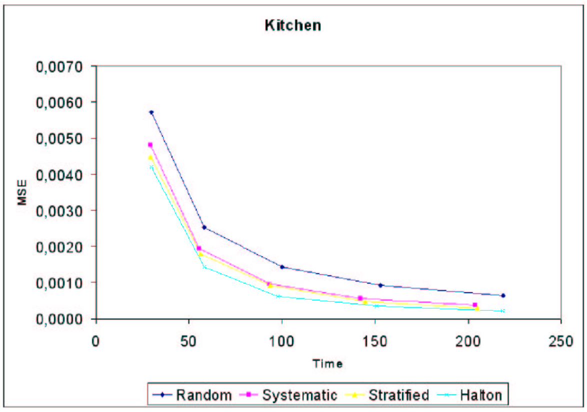
(b.i) Indirect illumination of Kitchen scene.

(b.ii) Obscurances of Kitchen scene.

Figure 1: These images show the two virtual models, a box (row a) and a kitchen (row b), used for the purpose of our comparison. In column i, we can see the diffuse indirect light computed with obscurances in both models. In column ii, pure obscurances are shown.

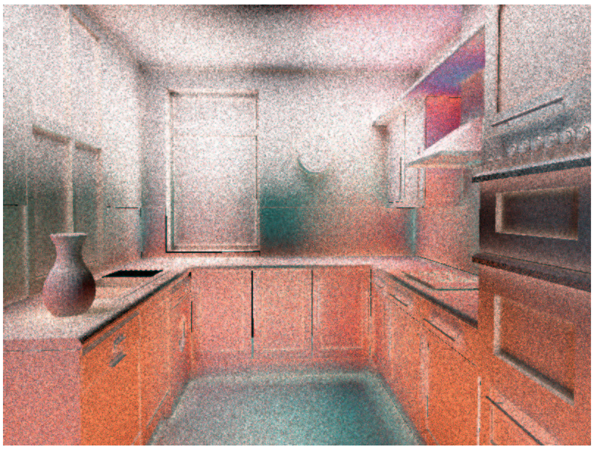


(a) Chart of Box scene.

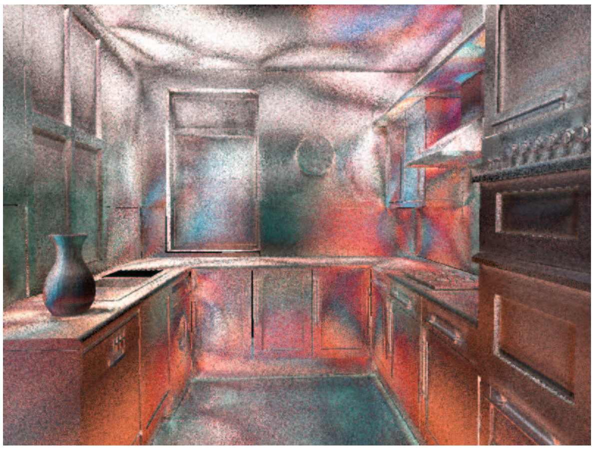


(b) Chart of Kitchen scene.

Figure 2: Charts of the comparison of efficiency for both box and kitchen models. Computation time for an image in seconds is measured in X axis, and Y axis measures the Mean Square Error, averaged for all pixels, for the computation of the obscurances. Series of 4, 9, 16, 25 and 36 samples, are measured for each technique and model.

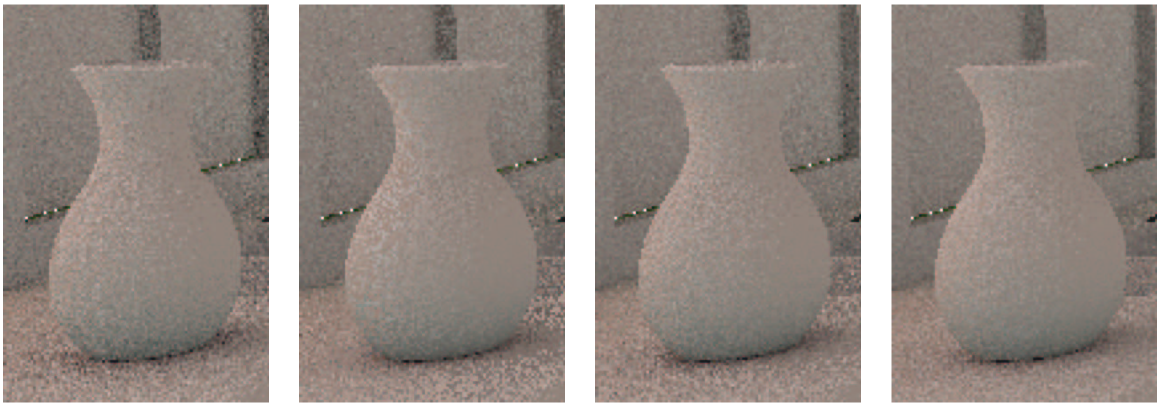


(a) MSE map for random.



(b) MSE map for systematic.

Figure 3: These images represent maps of Mean Square Error (MSE) for (a) uniform random and (b) systematic sampling obscurances. Systematic variation of error in (b) is clearly appreciated.



(a) Random. (b) Systematic. (c) Stratified. (d) Halton.

Figure 4: Detail from the kitchen model image, for a visual comparison of all four sampling methods; (a) uniform random, (b) systematic, (c) stratified and (d) quasi-Monte Carlo with random offset.

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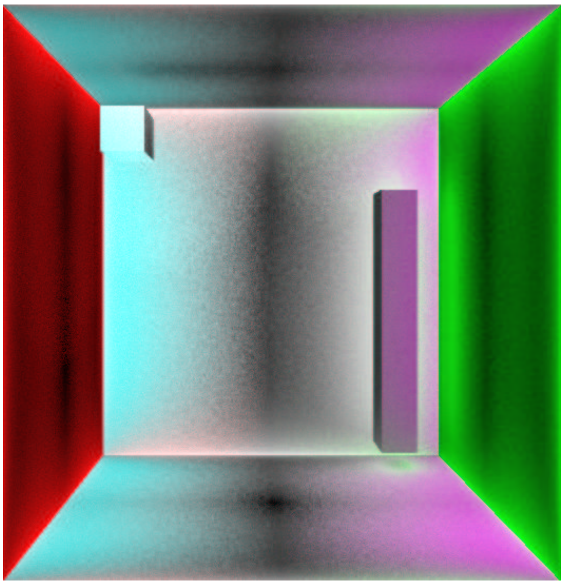

Figure 5: Color map for the sample variances for the obscurance computation. We can appreciate that different parts of the image show different variance values. Thus we conclude that adaptive sampling would lead to noticeable improvement in efficiency.

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